

Spatiotemporal control of charge +1 topological defects in polar active matter

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Topological defects are a conspicuous feature of active liquid crystals that have been associated with important morphogenetic transitions in organismal development. Robust development thus requires a tight control of the motion and placement of topological defects. In this article, we study a mechanism to control +1 topological defects in an active polar fluid confined to a disk. If activity is localized in an annulus within the disk, the defect moves on a circular trajectory around the center of the disk. Using an *ansatz* for the polar field, we determine the dependence of the angular speed and the circle radius on the boundary orientation of the polar field and the active annulus. Using a proportional-integral controller, we guide the defect along complex trajectories by changing the active annulus size and the boundary orientation.

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I. INTRODUCTION

Active stress generated by the transformation of chemical energy into mechanical work is at the base of fascinating autonomous movements seen in biological systems [1,2]. At the molecular level, it results from the interaction of molecular motors with filamentous protein assemblies. Due to their molecular form, active stress in living matter is often anisotropic. The principal direction of the stress typically coincides with orientational order exhibited by the biological material. Macroscopic orientational order can result, for example, from the alignment of filaments [3] or of elongated cells [4,5]. If individual filaments or cells are polarized, their alignment can result in polar or nematic orientational order.

Topological point defects are increasingly recognized to play a central role in organizing mechanical stress in biological systems. At such defects, the orientational order field is ill-defined. They are characterized by their topological charge, which can be either half-integer or integer for nematic order, whereas only integer charges occur for polar order. Defects have been reported to determine the position of cell death and extrusion [6], direct the formation of cell mounds [7–9], and are essential for surface deformations that accompany the

formation of mouth, foot, and tentacles during regeneration of the freshwater polyp *Hydra vulgaris* [10,11]. Activity-induced surface deformations orchestrated by topological defects also hold promises in the design of soft robots [12].

Topological defects of charge +1 play a particular role in surface deformation. These defects are unique in that they have continuous rotational symmetry. A homogeneous local rotation of the orientation field, referred to as a change in phase, causes a +1 defect to transition between aster, spiral, and vortex. For any other defect, a change in phase is simply associated with a global rotation around the defect center. Active defects of charge +1 can deform surfaces into different shapes ranging from domes to saddles depending on the phase of the defect [13]. Thus, +1 defects offer the possibility to control shapes by changing their phase. Phase changes can be performed in a restricted region around the defect, whereas the alternative, changing the topological charge of a defect, requires global changes of the orientational order.

It is thus desirable for biological and engineered systems to be able to control the arrangement of topological defects. Several proposals have been made to achieve spatiotemporal control of defect patterns in active nematics [14–20]. Such ideas have been realized in experiments on reconstituted cytoskeletal motor-filament systems [15,21]. Similar activity patterning techniques have been shown to be applicable to Toner-Tu polar systems [22]. Defects of charge +1 are rotationally symmetric and, in contrast to half-integer defects, do not self-propel. Persistent motion of integer defects was achieved using dynamic activity patterns [22]. In spite of these efforts, achieving controlled positioning and persistent motion of +1 defects remains challenging. We explore how symmetry

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breaking and heterogeneous activity can work together for a precise control of such defects.

In this work, we focus on a +1 topological defect in an active polar fluid confined to a disk. We show how heterogeneous activity induces defect self-propulsion and explain its physical origin. We then leverage this mechanism for spatiotemporal defect control using a closed-loop controller. We achieve precise radial and azimuthal motion and discuss potential applications.

II. DEFECT DYNAMICS ON A DISK

In the following, we briefly recall the hydrodynamic equations of an active polar fluid, where the magnitude of the polar order is not fixed. We then consider the steady state of such a fluid confined to a disk, where the boundary conditions impose a total defect charge of +1. Whereas the defect colocalizes with the disk center for homogeneous activity, it can circulate around the center if the fluid is passivated in the disk's central region.

A. Hydrodynamics of an active polar fluid

We investigate an incompressible two-dimensional (2D) polar active fluid confined to a disk using generalized hydrodynamics [23–25]. The polar field is described by the vector $\mathbf{p}$, which is associated with the free energy $F = \int f dA$, where

$$f = \frac{K_S}{2}(\nabla \cdot \mathbf{p})^2 + \frac{K_B}{2}(\nabla \times \mathbf{p})^2 + \frac{\chi}{2} \left(\frac{|\mathbf{p}|^4}{2} - |\mathbf{p}|^2 \right). \quad (1)$$

The first two terms describe how the free energy depends on splay and bend distortions of $\mathbf{p}$ with respective elastic coefficients K_S and K_B . The parameter χ controls the magnitude of a soft constraint for $|\mathbf{p}|^2 = 1$. We will refer to $|\mathbf{p}|$ as the polar order parameter.

The dynamics of $\mathbf{p}$ are governed by

$$\frac{D}{Dt} p_\alpha = \frac{1}{\gamma} h_\alpha + \lambda p_\alpha \Delta \mu - \nu u_{\alpha\beta} p_\beta. \quad (2)$$

In this expression, Greek indices go from 1 to 2 and we adopt the convention of summing over repeated indices. The molecular field $h_\alpha = -\frac{\delta F}{\delta p_\alpha}$ describes the response of p_α to deviations from its equilibrium configuration and the parameter γ represents rotational viscosity. Activity results from chemical processes driven by the chemical potential difference $\Delta \mu$. It couples directly to the dynamics of the polarization via the phenomenological coefficient λ . Possible molecular mechanisms leading to such a term include cytoskeletal filament disassembly, which involves hydrolysis of nucleotide triphosphates, and upstream activation or deactivation of regulators of filament assembly, for example, by small GTPases. Note that, for $\lambda \Delta \mu < 0$, activity tends to decrease polar order. The parameter ν is the flow alignment coefficient, which couples the polarization angle to the strain-rate tensor $u_{\alpha\beta} = \frac{1}{2}(\partial_\alpha v_\beta + \partial_\beta v_\alpha)$, where $\mathbf{v}$ is the velocity field. The corotational material derivative is given by $Dp_\alpha/Dt = \partial_t p_\alpha + v_\gamma \partial_\gamma p_\alpha + \omega_{\alpha\beta} p_\beta$, with the vorticity tensor $\omega_{\alpha\beta} = \frac{1}{2}(\partial_\alpha v_\beta - \partial_\beta v_\alpha)$.

We consider flows with low Reynolds number and assume that there are no external forces. Force balance then reads

$$\partial_\beta \sigma_{\alpha\beta} = 0, \quad (3)$$

for $\alpha = 1, 2$, where the total stress tensor can be decomposed into a symmetric and an antisymmetric part $\sigma_{\alpha\beta} = \sigma_{\alpha\beta}^{\text{sym}} + \sigma_{\alpha\beta}^{\text{ant}}$. The symmetric part of the total stress has components from the Ericksen, viscous, (symmetric) reactive, and active stress: $\sigma_{\alpha\beta}^{\text{sym}} = \sigma_{\alpha\beta}^{e,\text{sym}} + \sigma_{\alpha\beta}^v + \sigma_{\alpha\beta}^r + \sigma_{\alpha\beta}^a$. Here, the total Ericksen or elastic stress is given by

$$\sigma_{\alpha\beta}^e = -P\delta_{\alpha\beta} - \frac{\partial f}{\partial(\partial_\beta p_\gamma)} \partial_\alpha p_\gamma, \quad (4)$$

where P is the hydrostatic pressure. It is further decomposed into a symmetric and an antisymmetric part. The symmetric part $\sigma_{\alpha\beta}^{e,\text{sym}} = \frac{1}{2}(\sigma_{\alpha\beta}^e + \sigma_{\beta\alpha}^e)$ generalizes the hydrostatic pressure in the presence of the polarization field $\mathbf{p}$. The total antisymmetric stress reads

$$\sigma_{\alpha\beta}^{\text{ant}} = (p_\alpha h_\beta - p_\beta h_\alpha)/2 + \sigma_{\alpha\beta}^{e,\text{ant}}, \quad (5)$$

where $\sigma_{\alpha\beta}^{e,\text{ant}} = \frac{1}{2}(\sigma_{\alpha\beta}^e - \sigma_{\beta\alpha}^e)$ is the antisymmetric part of the Ericksen stress. The antisymmetric part of the total stress contributes to the angular momentum balance by mediating exchange between orbital and internal angular momentum. The remaining terms for viscous, reactive, and active stress components, respectively, are given by

$$\sigma_{\alpha\beta}^v = 2\eta u_{\alpha\beta}, \quad (6)$$

$$\sigma_{\alpha\beta}^r = \frac{\nu}{2}(p_\alpha h_\beta + p_\beta h_\alpha - p_\gamma h_\gamma \delta_{\alpha\beta}), \quad (7)$$

$$\sigma_{\alpha\beta}^a = -\zeta \Delta \mu (p_\alpha p_\beta - \frac{1}{2} p_\gamma p_\gamma \delta_{\alpha\beta}). \quad (8)$$

Here, η is the fluid's viscosity and ν is the flow alignment parameter. The parameter ζ couples the active chemical processes to the mechanics. The equations are closed by the incompressibility condition $\partial_\gamma v_\gamma = 0$, which determines P .

In the disk geometry, it is convenient to express the polar field $\mathbf{p}$ by its norm, that is, the polar order parameter $|\mathbf{p}|$, and the orientation angle ψ between $\mathbf{p}$ and the inward radial direction, $\cos(\psi) = -\hat{\mathbf{p}} \cdot \hat{\mathbf{r}}$, where a hat indicates a normalized vector. At the disk boundary $r = r_b$, we implement no-slip conditions for the velocity and impose $\psi = \psi_b$ as well as $|\mathbf{p}| = 1$. These boundary conditions impose a topological defect charge +1 in the polar field.

In the following, we scale time by γ/χ , length by $\sqrt{K_S/\chi}$, and energy by K_S . This leaves us with the following dimensionless parameters: $\tilde{\zeta} = \zeta \Delta \mu / \chi$, $\tilde{\lambda} = \gamma \lambda \Delta \mu / \chi$, $\tilde{\eta} = \eta / \gamma$, and $\tilde{K}_B = K_B / K_S$. Note that both, $\tilde{\zeta}$ and $\tilde{\lambda}$, depend on the chemical potential difference $\Delta \mu$. In experiments, it is easier to change $\Delta \mu$ than ζ or λ . For this reason, $\Delta \mu$ is varied while keeping $\tilde{\lambda}/\tilde{\zeta} = -7.5$ constant unless stated otherwise, and results are shown in terms of $\tilde{\lambda}$.

For numerical solutions to the dynamic equations, we used a randomized point cloud in space and evaluated the differential operators using discretization-corrected particle strength exchange (DC-PSE) [26]. The scheme was implemented in the OpenFPM library for scalable scientific computing [27] using a template-expression system for hydrodynamic equations [28]. We solved for Stokes flow with

TABLE I. Values of the nondimensional parameters used in the main text, unless stated otherwise.

$\bar{\eta}$	2
γ/χ	1
$\sqrt{K_S/\chi}$	$\sqrt{10}$
$\bar{\lambda}$	$[-0.9, 0]$
$\bar{\zeta}$	$-\bar{\lambda}/7.5$
$\bar{K}_B$	1
r_b	25
r_i	$[0, 0.8]r_b$
ν	2

a pressure-correction algorithm to ensure incompressibility [29] and evolved the system in time using the adaptive Adams-Bashforth-Moulton time-stepping method [30]. See Appendix A for details and Table I for parameter values.

B. Spontaneous defect motion

In this section, we study the dynamic equations (2) and (3) for extensile active stress, $\bar{\zeta} \geq 0$, which is spatially homogeneous in the region $r \geq r_i$ and vanishes for $r < r_i$. We briefly discuss the case $r_i = 0$ and then consider the general case.

1. Homogeneous activity

For homogeneous extensile activity, we obtain rotating spiral defects if the activity exceeds a critical value (Appendix B). This is in agreement with previous reports [23], where the polar order parameter was kept constant throughout. Although in the present system $|\mathbf{p}|$ is not fixed, in thermodynamic equilibrium, the free energy (1) still yields $|\mathbf{p}| = 1$ away from defects. Activity, however, modifies the magnitude of the polar order. Specifically, in a steady state with $\mathbf{v} = 0$ and $|\mathbf{p}| = \text{const}$, the active term in Eq. (2) leads to

$$|\mathbf{p}|^2 = 1 + \bar{\lambda} \equiv P_0^2. \quad (9)$$

In the presence of a flow, the polar order parameter is further affected by flow alignment (see Appendix B).

2. Heterogeneous activity

We next investigate the dynamics if $r_i > 0$, such that the system is passive in a central disk and activity is confined to an annulus extending to the boundary of the disk [Fig. 1(a)]. From now on, we use the one-constant approximation, $K_B = K_S$, and set the boundary angle to $\psi_b = \pi/4$.

For an active stress that is larger than a critical value, $\bar{\lambda}_c < \bar{\lambda} < 0$, the defect stays in the passive region and moves to the center of the disk. The defect adopts a spiral configuration [Fig. 1(b)] and generates a rotationally symmetric flow similar to the homogeneous case [see Appendix B, Figs. 6(c) and 6(d)]. For $\bar{\lambda} < \bar{\lambda}_c$ and $\bar{\zeta} > 0$, the defect leaves the passive region and settles on a circular trajectory around the center [Fig. 1(c)]. The corresponding angular speed ω and radius r_d increase with the activity. Let us note that the defect remains in the disk center if $\bar{\lambda} \geq 0$.

For a defect circulating around the disk center, the polarization field is not rotationally symmetric around the defect,

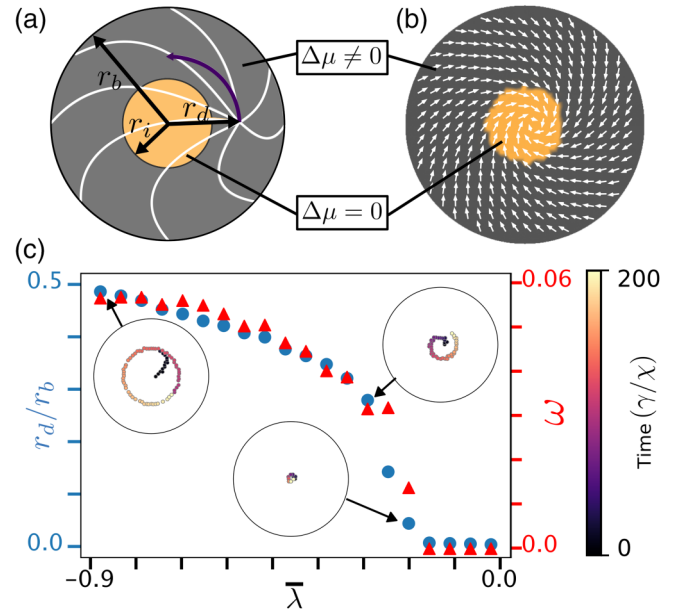


FIG. 1. Defect on a disk with heterogeneous activity pattern. (a) Illustration of the system. At the disk boundary, $r = r_b$, we impose $|\mathbf{p}| = 1$ and $\psi_b = \pi/4$. Gray: active region, $\bar{\zeta}, \bar{\lambda} \neq 0$; yellow: passive region, $\bar{\zeta} = \bar{\lambda} = 0$. White lines: polarization field lines. Dark purple curved arrow: circular defect path with radius r_d . (b) Initial configuration of the numerical solver. White arrows: polarization field, gray and yellow regions as in panel (a). (c) Radial defect position r_d (blue dots) and angular defect speed ω (red triangles) as a function of activity $\bar{\lambda}$. Insets: Trajectories of the defect for $\bar{\lambda} = -0.857$ (left), $\bar{\lambda} = -0.300$ (right), and $\bar{\lambda} = -0.214$ (center). Colors: time along trajectories. Parameters as in Table I and $\bar{\lambda} = -7.5\bar{\zeta}$ and $r_i/r_b = 8/25$.

even in its vicinity [Fig. 2(a)]. Instead, it shows distinct areas of increased bend or splay, where splay is increased in front of the circulating defect and bend behind [see insets in Fig. 2(a)]. The polar order parameter is larger in the passive than in the active region, as expected from Eq. (9). Moreover, in the passive region, the polarization field approaches a uniform configuration. Still, flows are also present in the passive region [Fig. 2(b)]. As in the case of homogeneous activity, these flows are driven by the active stress σ^a and active contributions to the dynamics of $\mathbf{p}$ through λ .

The flow speed is largest close to the defect and decreases toward the passive region and also toward the disk boundary, where we impose $\mathbf{v} = 0$. At the defect position, the flow field is purely azimuthal, and its angular speed is similar to that of the defect. In the following, we estimate the angular defect velocity ω through an *ansatz* for the polarization field.

III. CONTROL OF DEFECT MOTION

In the previous section, we found that +1 topological defects circulate around the center of a disk that is divided into a passive central disk and an outer active annulus. In the following, we introduce an *ansatz* for the polarization field. Based on this *ansatz*, we find that the radius r_i of the passive innerdisk and the boundary angle ψ_b can be used as parameters for controlling the defect. Finally, we implement a

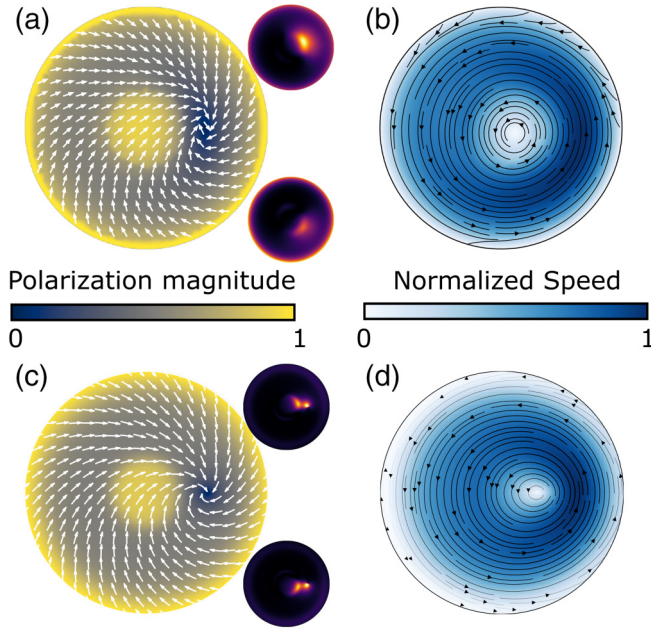


FIG. 2. Polarization and flow fields for a defect circulating with constant angular velocity. (a), (b) Numerical solution of the dynamic equations. (a) White arrows: $\hat{\mathbf{p}}$, colors: $|\mathbf{p}|$. Insets: Splay (top) and bend distortions (bottom) in $\mathbf{p}$. Hot colors indicate stronger deviations from the equilibrium configuration. (b) Black lines: streamlines; colors: speed normalized to its maximum $v_{\max} = 1.14$. Parameters as in Fig. 1(c) and $\bar{\lambda} = -0.857$. Polarization [panel (c)] and flow fields [panel (d)] given by the *ansatz* from Sec. III A. Parameters: $\bar{p}_0 = 0.894$, $\Delta P_1 = 0.383$, $\Delta P_2 = 0.623$, $r_1 = 0.038$, $r_2 = 0.104$, $\lambda_1 = 0.037$, and $\lambda_2 = 0.08$. Furthermore, $v_{\max} = 1.07$.

proportional-integral (PI) controller to simultaneously modify ψ_b and r_i to set the defect on a prescribed path.

A. An approximate solution for the defect dynamics

To start, we notice that the orientation of the polar field varies weakly along the radial directions from the defect position to the boundary [Fig. 2(a)]. In our *ansatz* for $\mathbf{p}$, we choose the orientation to be constant along the radial directions from the defect center. As a consequence, the polarization field is fully determined by the boundary angle ψ_b and by the radial defect position r_d .

We use polar coordinates (r, θ) centered at the disk center and consider a defect located at (r_d, θ_d) . A convenient representation of the polarization field is obtained by introducing a second polar coordinate system (ρ, ϕ) centered at the defect. The coordinate systems are oriented such that $\phi = 0$ corresponds to the direction θ_d . Without loss of generality, we set $\theta_d = 0$. For a point with coordinates (r, θ) in the disk-centered system and the coordinates (ρ, ϕ) in the defect-centered system, we have

$$\rho = \sqrt{r^2 - r_d^2 \sin^2 \phi} - r_d \cos \phi, \quad (10)$$

$$\tan \theta = \frac{\rho \sin \phi}{r_d + \rho \cos \phi}. \quad (11)$$

From the above expressions, we obtain the polarization angle $\Psi(\phi)$ along the direction ϕ . It is given by the boundary

angle ψ_b measured relative to the radial direction from the disk center by

$$\Psi(\phi) = \psi_b + \theta_b(\phi) - \phi. \quad (12)$$

Here, $\theta_b(\phi)$ is given by Eqs. (10) and (11) with $r = r_b$ and, in the disk-centered system, corresponds to the polar angle of the boundary point associated with direction ϕ in the defect-centered system.

It remains to specify the polar order parameter, which should account for two distinct features. First, the polar order is high in the passive center and close to the disk boundary, but low in an annulus between these two regions [Fig. 2(a)]. Second, polar order decreases when approaching a defect. We use

$$|\mathbf{p}| = p_0(r)(1 - \exp\{-\rho/\varepsilon\}). \quad (13)$$

Here, the second factor approximates the polar order parameter around a defect. The characteristic length ε is thus determined by the elastic parameters K and χ [Eq. (1)], $\varepsilon^2 \sim K/\chi$. The factor p_0 captures how activity modulates the order parameter. Explicitly, we write

$$p_0 = \bar{p}_0 + \frac{\Delta P_1}{1 + \exp[-(r - (r_i - r_1))/\lambda_1]} + \frac{\Delta P_2}{1 + \exp[-(r - (r_b - r_2))/\lambda_2]}. \quad (14)$$

For $\Delta P_1 = \Delta P_2 = 0$, the profile is constant, and the parameter $\bar{p}_0$ sets the steady-state value in a passive system. The two additional terms are step functions that enable spatial modulation of the order parameter at the center, near the disk boundary, and in the intermediate region. The first term modifies the polar order parameter outside the passive central region, $r > r_i - r_1$, while the second acts closer to the boundary, $r > r_b - r_2$. These contributions adjust the polar order by ΔP_1 and ΔP_2 , respectively. The parameters $r_{1,2}$ control the sizes of the central and boundary regions, and $\lambda_{1,2}$ determine the sharpness of the transitions between them.

Our *ansatz* for the polarization field is qualitatively in good agreement with the numerical solution [Fig. 2(c)]. In particular, it captures the bend and splay distortions in the front and the back of the defect. Other functional forms or a larger number of fit parameters could be used to get a better quantitative agreement between the *ansatz* and the real solution. However, the main goal of the present analysis is to provide qualitative insight into defect motion rather than a quantitative fit to the numerical solution.

Given the polarization field, which includes, in particular, the radial defect position, we obtain the flow field from force balance [Eq. (3)]. It is equivalent to the Stokes equation, with a force density $\nabla \cdot (\sigma - \sigma^v)$ and can be solved using the corresponding Green's function (Appendix C). It turns out that the force density is dominated by the reactive and active components, such that we only use $\nabla \cdot (\sigma^a + \sigma^r)$ in the following. Whereas the contribution of σ^a can be readily obtained analytically, the computation of the contribution of σ^r is more involved. We therefore compute both numerically and note merely that the analytics shows that the active force $\nabla \cdot \sigma^a$ is directed parallel or antiparallel to $(\cos(2\psi_b), \sin(2\psi_b))$. Numerically, we find the same to be true for the contribution from the reactive stress.

TABLE II. Timing and values of changes in boundary angle to generate Fig. 3(c), $\bar{\xi} = 0.114$.

Time (s)	ψ_b
0–50	0.25π
50–100	0π
100–150	-0.25π
150–200	0π
200–250	0.25π

Visual inspection indicates that the *ansatz* for the polarization field, combined with the approximation that retains only the reactive and active stress contributions, provides a reasonable estimate of the flow field [Fig. 2(d)]. The main difference compared to the full solution for the velocity field is that the minimum of the approximate velocity occurs farther from the disk center.

B. Azimuthal motion of the defect

For steady defect motion around the disk, we observe a rigid-bodylike rotation of the fields (see Fig. 2). This implies that the system has a constant, nonzero angular momentum given by $L = \int \mathbf{r} \times \mathbf{v} dA$. Note that the total angular momentum, which includes components resulting from the polarization field, vanishes. The nonzero angular momentum L must arise from a net torque on the fluid, which can be written as $T = \int \mathbf{r} \times \mathbf{f} dA$, where $\mathbf{f}$ are the forces driving the fluid motion. As outlined above, the force density is dominated by the reactive and active components; hence, we can estimate the total torque on the system from $T = \int \mathbf{r} \times \nabla \cdot (\sigma^a + \sigma^r) dA$. Using our *ansatz*, we numerically estimate the total torque on the system and get an estimate for $\omega \propto T$ [Fig. 3(a)]. This shows a $\sin(2\psi_b)$ relationship that agrees well with the numerical results [Fig. 3(a)]. On the contrary, the numerical results show that varying the boundary angle has a much smaller effect on the radial position of the defect [Fig. 3(b)].

We can leverage the dependence of the motion of the defect on the boundary angle ψ_b to dynamically control the defect's position in the disk. By alternating ψ_b between $\pm\pi/4$, we cause the defect to propagate back and forth azimuthally at maximal speed and constant radius, resulting in an arc [Fig. 3(c)]. The length of the individual arcs can be controlled

TABLE III. Timing and values of changes in boundary angle to generate Fig. 3(d), $\bar{\xi} = 0.114$.

Time (s)	ψ_b
0–20	0.2π
20–60	0.7π
60–100	0.2π
100–140	0.7π
140–180	0.2π
180–220	0.7π
220–160	0.2π

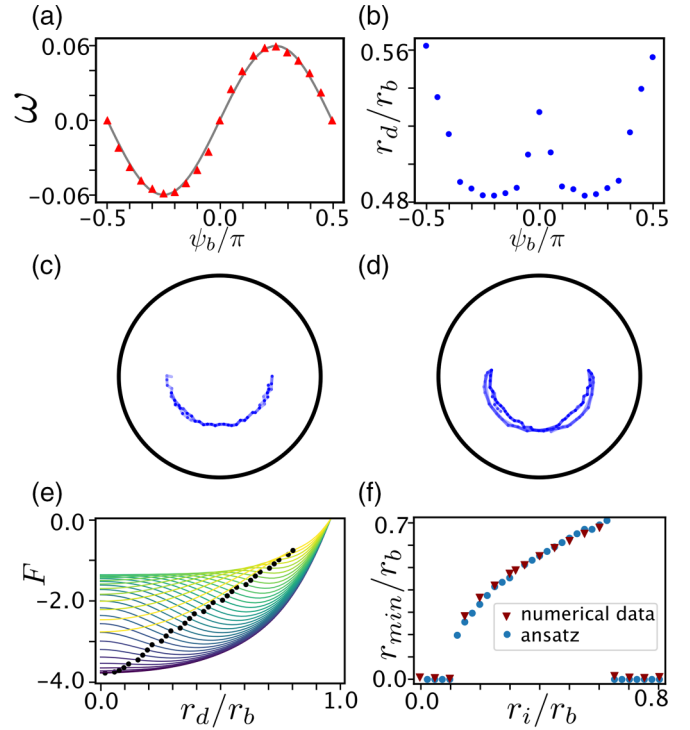


FIG. 3. Controlling defect motion through the boundary angle. (a) Angular defect speed ω as a function of the boundary angle ψ_b . Red triangles: numerics; gray line: from *ansatz*. (b) Radial defect position r_d as a function of ψ_b . (c) Defect trajectory for ψ_b changing according to Table II. (d) Defect trajectory for ψ_b changing according to Table III. (e) Free energy F as a function of the radial defect position r_d for different radii of the passive region $r_i/r_b \in \{0, 0.8\}$ indicated by black dots using the *ansatz* for the polarization field. (f) Position of the elastic potential's minimum as a function of r_i . Red triangles: numerics; blue dots: *ansatz*. Parameters as in Table I and $\bar{\lambda} = -0.857$.

by adjusting the timing over which ψ_b is varied; the detailed time course of ψ_b for Fig. 3(e) is given in Table II.

We can also utilize the dependence of the radial position of the defect on ψ_b to generate a 2D path. For example, if we periodically switch ψ_b between $(0.45 \pm 0.25)\pi$ [Table III], we introduce a radial component of $\mathbf{f}$. This leads to the defect tracing a figure eight pattern and shows the potential of this technique [Fig. 3(d)].

C. Controlling the radial defect position

Using only the boundary angle ψ_b to guide the defect has two key weaknesses. First, the azimuthal motion is coupled to the radial position; thus, one cannot control them independently. Second, we can only access a narrow range of radii [Fig. 3(b)].

We now show that the defect's radial position is determined predominantly through elastic interactions that are independent of ψ_b . To this end, we evaluate the free energy [Eq. (1)] for the *ansatz* polarization field (13). Note that through the dependence on the radius r_i of the passive central disk as well as on $\Delta P_{1,2}$, the results based on the *ansatz* also depend on activity.

In Fig. 3(e), we show the free energy as a function of the defect position r_d for different values of the radius r_i . For sufficiently small values of r_i , the energy exhibits a single minimum at $r_{\min} = r_d$. When r_i falls below a critical value, this minimum is localized at the disk center, $r_{\min} = 0$. Above the critical value, $r_{\min}$ increases monotonically with r_i [Fig. 3(f)]. Within our numerical accuracy, the transition appears to be continuous.

The radial defect positions obtained from our *ansatz* agree very well with the numerical solutions [Fig. 3(f)]. This leads to an intuitive picture of the underlying physical mechanism. Within the *ansatz*, the value of $r_{\min}$ is set by the competition between two effects. First, since the polar order is small at the defect but fixed to unity at the boundary, gradient contributions to the free energy increase as the defect is positioned closer to the boundary. This effectively introduces a repulsive force between the boundary and the defect but fixed to unity at the boundary. As a consequence, the defect is pushed toward the disk center. Second, the term proportional to χ in the free energy becomes important when the defect is located in the passive region. As soon as the passive region is large enough, the gain in energy by moving the defect further to the center and thus reducing the gradients in $\mathbf{p}$ is outweighed by the increase in energy due to the deviation of the order parameter from its preferred value P_0 . Balancing both contributions leads to $r_d > 0$ for sufficiently large r_i .

As the size of the passive disk reaches a second critical value, the energy is essentially the same as for a homogeneous passive system as long as r_d is sufficiently small. As a consequence, the energy exhibits a second minimum at $r_d = 0$ in this case. While this suggests coexistence of two steady states, we only observe the global minimum numerically.

D. Independent control of angular speed and radius

As we have seen, the boundary angle ψ_b can be used to set the angular defect velocity and the radial defect position. To reach independent control of both quantities, we utilize the radius r_i , which only affects the radial defect position. Specifically, we chose to implement a feedback controller that modulates r_i over time to target a specific radial position of the defect.

We opt for a proportional-integral controller [31]. We define a target radius for the defect, here denoted by r_d^* . The error in the target observable is calculated as $r_d - r_d^*$, and the control parameter r_i is updated proportional to the instantaneous and the time-integrated error [Fig. 4(a)]. This approach allows us to avoid asymptotic deviations from the target value at steady state [31]. The parameters of the PI controller must be carefully tuned to achieve the desired control (Appendix A3) that is robust to changes in activity (Fig. 5).

In Fig. 4(b), we demonstrate the efficacy of controlling the radial position of the defect with the PI controller. Changes in the radial position r_d following changes in ψ_b are rapidly compensated by the PI controller [Fig. 4(b)]. When changing the target radius r_d^* , the direction of circular defect motion remains unchanged, whereas its speed changes. This demonstrates that we can independently control the defect's radial position r_d and its angular speed by using the PI controller and adjusting the boundary angle ψ_b .

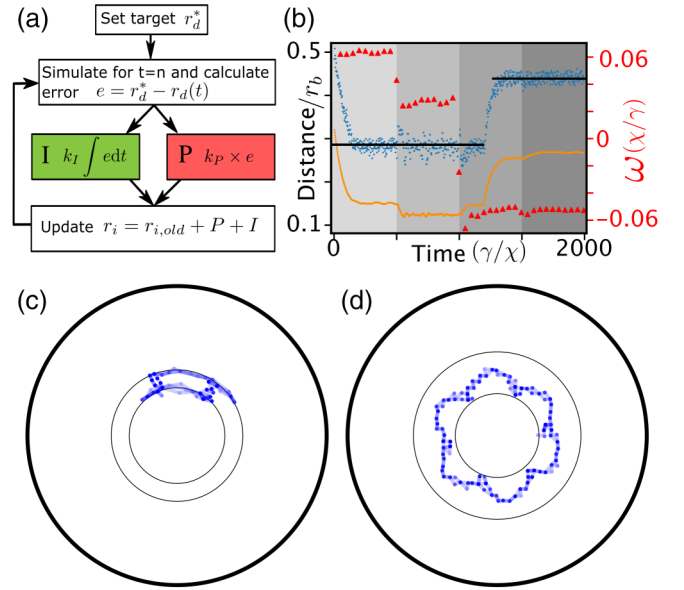


FIG. 4. Dynamic feedback control of the defect location. (a) Schematic of the PI controller. The controller maintains the radial distance of the defect at a desired setpoint r_d^* by dynamically adjusting the radius of the passive core, r_i . For each r_i , the system is simulated for $n = 10$ time steps to determine the resulting r_d , which yields the error that is fed back into the controller. The new r_i is calculated as $r_i = r_{i,old} + P + I$, where P denotes the proportional part and I the integral part of the controller. (b) Independent control of defect radius r_d and defect angular velocity ω . The radial setpoint changes from $r_d^*/r_b = 7/25$ to $r_d^*/r_b = 11/25$ at time step 1200 (black lines). Meanwhile, the boundary angle ψ_b increases each $500\gamma/\chi$ by $\pi/5$ starting from $\psi_b = \pi/4$. Blue dots: actual radial defect position; red triangles: angular velocity, orange dots: r_i . Parameter values as in Table I and $\bar{\lambda} = -0.857$. (c) Defect trajectory (blue dots) when prescribing a closed loop with right-angle corners. Static parameter values as in Table I and $\bar{\lambda} = -0.857$; dynamic parameters: Table IV. Control parameters: $k_P = 5 \times 10^{-4}$ and $k_I = 5 \times 10^{-3}$. (d) Defect trajectory (blue dots) when prescribing a hanagata shape with internal angles of π . Static parameter values as in Table I and $\bar{\lambda} = -0.514$; dynamic parameters: Table V. In panels (c) and (d), gray circles correspond to the extreme values of r_d^* .

The enhanced control of the defect made possible by the PI controller allows us to generate more complex patterns that were previously not possible. We demonstrate this by generating a closed loop with right-angle corners [Fig. 4(c) and Table IV] and a hanagata shape [Fig. 4(d) and Table V]. Even more complex patterns would be possible using appropriate sequences of target radii and boundary conditions.

IV. DISCUSSION

In summary, we investigated the behavior of active polar fluids confined to a 2D disk with a single topologically constrained $+1$ defect. For a heterogeneous activity pattern, the defect moves in a circle around the center. The circular motion depends on the boundary angle of the polar field and the size of the active annulus. Based on these dependencies, we imposed target trajectories on the defect by means of a PI controller.

TABLE IV. Timing and values of changes in boundary angle and target radius to generate Fig. 4(c), $\bar{\zeta} = 0.114$.

Time (s)	ψ_b	r_d^*/r_b
0–20	0.0π	0.44
20–40	0.13π	0.44
40–80	0π	0.44
80–420	0π	0.32
420–440	-0.13π	0.32
440–480	0π	0.32
480–620	0π	0.44
620–640	0.13π	0.44
640–685	0π	0.44
685–820	0π	0.32
820–840	-0.13π	0.32
840–885	0π	0.32
885–1020	0π	0.44
1020–1040	0.13π	0.44
1040–1090	0π	0.44
1090–1220	0π	0.32
1220–1240	-0.13π	0.32
1240–1400	0π	0.32

Combining heterogeneous activity patterns with changes of the boundary conditions gives ample possibilities for controlling the motion of individual defects. Heterogeneous activity patterns different from the one studied in this work have been explored theoretically before. On a disk, complex activity patterns in combination with imposing vorticity were shown to set the sense of rotation of a pair of $+1/2$ defects [14]. Elementary activity patterns were identified to create, move, and braid $1/2$ defects [19]. These included $-1/2$ defects, which do not self-propel. By continuously updating activity patterns $+1$ defects, which do not self-propel either were moved in a polar Toner-Tu system [22].

TABLE V. Timing and values of changes in boundary angle and target radius to generate Fig. 4(d), $\bar{\zeta} = 0.069$.

Time (s)	ψ_b	r_d^*	Time (s)	ψ_b	r_d^*/r_b
0–60	0.016π	0.56	1080–1140	0.016π	0.56
60–120	0.016π	0.28	1140–1200	0.016π	0.28
120–180	0	0.28	1200–1260	0	0.28
180–240	0.016π	0.56	1260–1320	0.016π	0.56
240–300	0.016π	0.28	1320–1380	0.016π	0.28
300–360	0	0.28	1380–1440	0	0.28
360–420	0.016π	0.56	1440–1500	0.016π	0.56
420–480	0.016π	0.28	1500–1560	0.016π	0.28
480–540	0	0.28	1560–1620	0	0.28
540–600	0.016π	0.56	1620–1680	0.016π	0.56
600–660	0.016π	0.28	1680–1740	0.016π	0.28
660–720	0	0.28	1740–1800	0	0.28
720–780	0.016π	0.56	1800–1860	0.016π	0.56
780–840	0.016π	0.28	1860–1920	0.016π	0.28
840–900	0	0.28	1920–1980	0	0.28
900–960	0.016π	0.56	1980–2040	0.016π	0.56
960–1020	0.016π	0.28	2040–2100	0.016π	0.28
1020–1080	0	0.28	2100–2160	0	0.28

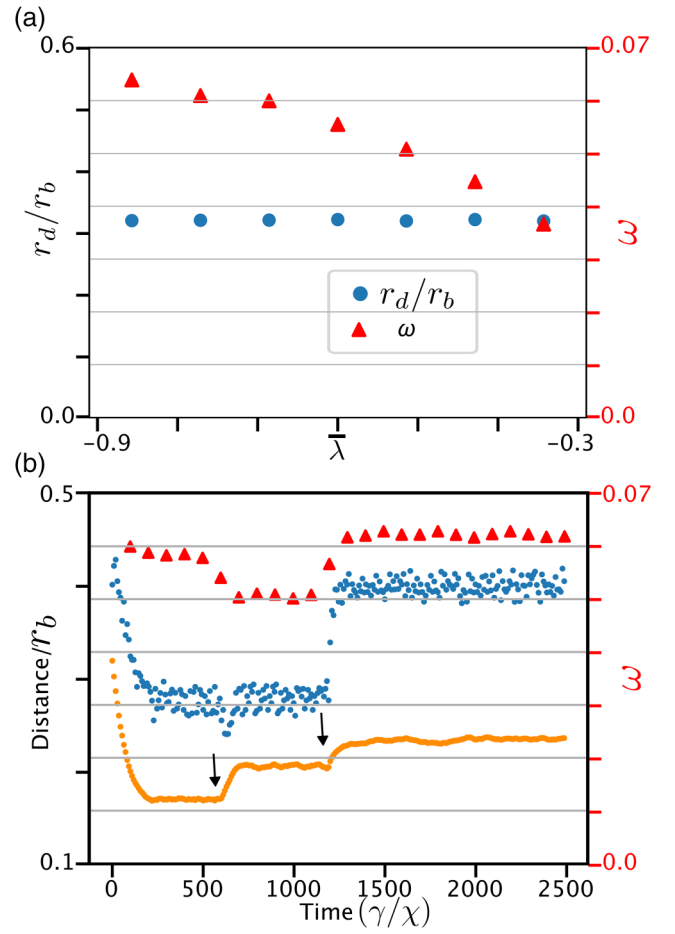


FIG. 5. (a) Angular velocity of the defect for varying activity while holding the radial position of the defect constant with the PI controller setpoint $r_d^*/r_b = 8/25$. (b) Controlled dynamics under abrupt change of r_d^*/r_b from $7/25$ to $10/25$ at time $1200\gamma/\chi$. At times $600\gamma/\chi$ and $1200\gamma/\chi$, the activity $\bar{\lambda}$ is changed, first from -0.686 to -0.514 , then to -0.857 , indicated by black arrows. This changes the angular defect velocity (red triangles, right scale). Throughout, the controller maintains the defect position near the setpoint.

The effects on the orientation of $+1/2$ defects through spatial jumps in activity similar to the border between the passive innercircle and the active annulus considered in the present work were explored in Ref. [18]. Reorientation of $+1/2$ defects was shown to be feasible by alignment with activity gradients [16,17].

Experimentally, defect control in active systems has been achieved in reconstituted motor-filament systems [15,21]. In these studies, light was used to increase motor activity in certain regions, which led to an accumulation of $+1/2$ defects and allowed the authors to bias the motion of single $+1/2$ defects along coarsely predefined paths [15]. A PI controller was implemented to control the coarse-grained flow speed of a microtubule-based nematic [21]. Other control mechanisms could be envisioned, for example, based on reinforcement learning as was used in a theoretical study to control $\pm 1/2$ defects [20].

The potential advantage of the system introduced in the present work is a higher precision of the control of defect

trajectories, which relies on the ability to set the boundary angle ψ_b . In reconstituted motor-filament systems, this might be challenging, although the capacity of magnetic fields to align microtubules when mixed with an appropriate synthetic nematic liquid crystal has been demonstrated experimentally [32]. For controlling cell monolayers, patterned surfaces provide promising alternatives [8,33–35]. While currently most patterns are static, new developments aim also at dynamic surface patterning [36]. Given the importance of topological defects in tissue morphogenesis [8,10,11], we consider this a particularly interesting research direction to follow.

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DATA AVAILABILITY

The code used for computing numerical solutions of the dynamic equations and the PI controller is openly available [37].

APPENDIX A: NUMERICS

1. Solution algorithm

Our custom simulation code is written in C++ using the OpenFPM library for scalable scientific computing [27]. For the discretization of the differential operators, DC-PSE [26, 38] is used, which is implemented in OpenFPM via a template-expression system for partial differential equations [28].

DC-PSE is a mesh-free collocation method, and discretization points do not need to lie on a grid. A (boxsize $\times$ boxsize) Cartesian grid is created as a template for the particles of DC-PSE, where boxsize = $2r_b$, but can be changed independently if higher resolution is wanted. A circular grid is added, whose center is defined at (boxsize/2, boxsize/2). The bulk particles are initially placed on a Cartesian grid and retained only if their positions lie within a radius, $r_b = 25 - 7\text{spacing}/6$ from the domain center. To avoid directional bias introduced by the underlying Cartesian grid, a uniformly distributed random perturbation in the interval $(-0.2/r_b, 0.2/r_b)$ is added independently to each coordinate. Without this perturbation, the discretization of the differential operators in the framework tends to favor gradients aligned with the Cartesian axes, producing nonphysical quadrangular flow patterns.

Additionally, particles near the domain boundary are classified as boundary particles at radius $r_b - 3\text{spacing}/4$. A corresponding ghost-particle layer is generated on the circular boundary at radius r_b by radially projecting nearby particles onto the outer boundary. These layers are used to impose Dirichlet anchoring conditions on the polarization field. At the boundary and ghost particles, the polarization is fixed to unit magnitude $|\mathbf{p}|^2 = 1$ and to a constant angle ψ_b , where ψ is defined with respect to the radial direction in the disk-centered

coordinate system. No-slip boundary conditions are imposed independently on the velocity field through the Stokes solver. The characteristic length scale for the polarity magnitude to go from 1 to 0 can be determined as $\varepsilon = \sqrt{K/\chi} = \sqrt{10}$ (see Table I for the physical parameters). To avoid finite-size effects, the radius of the disk r_b has to be chosen such that $r_b \gg \varepsilon$. Furthermore, the interaction cutoff radius rCut, which determines the neighborhood size used in the differential operator stencil, must resolve the polarization healing region. Therefore, $\text{rCut} \geq \varepsilon \times \text{spacing}$, corresponding to approximately $\sqrt{10} \times \text{spacing}$.

The polarity field is first relaxed to a stationary bulk configuration under the imposed boundary condition assuming vanishing flow. The resulting configuration is used as an initial condition for further time integration, which is realized by using the adaptive Adams-Bashforth-Moulton method. This is a multistep method combining an explicit predictor (Adams-Bashforth) with an implicit corrector (Adams-Moulton) step [30], from the `boost::numerics::odeint` library using their `make::controlled` option. The time tolerance and the relative time tolerance are both set to $7e-3$. This is combined with the OpenFPM methods `DCPSE_scheme` for discretizing spatial derivatives and `petsc_solver` for solving the Stokes problem. The incompressibility constraint $\partial_\gamma v_\gamma = 0$ is imposed by using the pressure-correction algorithm as previously described [29] at each simulation step. The velocity field itself is generated self-consistently through the active stresses arising from the polarization field and the no-slip hydrodynamic boundary conditions.

Defect tracking, output writing, and the PI controller, as well as changes in the activity, are implemented as `Observer` methods to the Adams-Bashforth-Moulton method. The defect is tracked as the local minimum in polarization magnitude, which is broadcast and compared across all compute processes to find the defect position for controller evaluation.

2. System parameters

As introduced before in Sec. II A, we use the energy scale K_S , the length scale $\sqrt{K_S/\chi}$, and the timescale γ/χ . We get the two activities $\tilde{\lambda} = \frac{\gamma\lambda\Delta\mu}{\chi}$ and $\tilde{\zeta} = \frac{\zeta\Delta\mu}{\chi}$ as well as the dimensionless viscosity $\tilde{\eta} = \eta/\gamma$. We introduce the dimensionless Frank constant $dK = \frac{K_B - K_S}{K_S}$, which captures whether the system favors bend or splay deformations in $\mathbf{p}$. Table I shows the values for the system parameters used in numerical simulations if not stated otherwise.

3. PI-controller configuration

To achieve stable control, the amplifications k_P and k_I must be chosen such that overshooting, oscillations, and transient errors are balanced. As expected, for $k_P \neq 0$ and $k_I = 0$ we observe an asymptotic steady-state error known as *droop* [21]. Introducing an integral part can reduce droop, but if k_I is too large the system is prone to overshooting. If k_P is too large, the system becomes oscillatory and eventually unstable. Values too small do not achieve the desired control. Here, we first set k_I to be small and decrease k_P from a large value until oscillations vanish. Then, k_I was subsequently increased, until droop was minimized. Using these $k_P = 5 \times$

10^{-4} and $k_I = 5 \times 10^{-3}$, we achieve stable control of r_d . Additionally, an antiwindup mechanism is introduced with limits $\pm 2\text{spacing}$.

a. Independence of radial control and activity

Figure 5(a) shows that for a range of activities, the defect position can be held at a constant $r_d/r_b = 8/25$ by the PI controller. Further, we can observe that the angular velocity of the defect reduces as activity is reduced for a constant r_d .

Figure 5(b) shows that the PI controller is able to maintain the defect's radial position during changes in the activity. The activity is reduced at time $600\gamma/\chi$, which prompts the PI controller to increase r_i to maintain r_d . When activity is reduced, the difference in the stable magnitude of the polar field between the active and passive areas is reduced, which in turn reduces the gradient of the elastic potential of the defect between these two regions; thus, r_i must increase to compensate. At time $1200\gamma/\chi$, the setpoint r_d^* and the activity are changed to $10/25$ and to $\tilde{\zeta} = 0.114$, respectively. The PI controller successfully regulates r_i to reach and stabilize the desired setpoint.

APPENDIX B: STEADY STATE FOR HOMOGENEOUS ACTIVITY

In this Appendix, we study the base case of homogeneous activity for different values of the elastic anisotropy $\overline{dK} = \overline{K}_B - 1$, $\bar{\lambda} = 0$, and a constant value of the angle ψ_b along the boundary. The latter condition implies the existence of topological defects with a total charge +1 (Fig. 6).

In the absence of active stress, $\tilde{\zeta} = 0$, the system equilibrates to $\mathbf{v} = 0$ and a polarization field $\mathbf{p}$ with $|\mathbf{p}|^2 = 1$. There is one defect of $\mathbf{p}$ situated in the disk center. The orientation of the equilibrium polarization field is governed by $\overline{dK}$. When bend deformations are energetically more costly than splay, $\overline{dK} > 0$, the angle ψ decreases monotonically toward zero at the disk center, corresponding to an aster defect [Fig. 6(a)]. Conversely, when bend is energetically favored, $\overline{dK} < 0$, ψ approaches $\pm\pi/2$ at the center, yielding a vortex defect [Fig. 6(b)]. Since the imposed boundary angle differs from these values, the steady-state polarization field smoothly transitions from an aster or vortex defect in the center to a spiral at the boundary.

For a contractile activity $\tilde{\zeta} = -1.6$, the steady state retains its rotational symmetry [Figs. 6(c) and 6(d)]. In contrast to the equilibrium case, however, it exhibits an azimuthal flow [Fig. 6(e)]. The flow is driven by gradients in the active stress resulting from the radial gradient in ψ . The flow alignment term in Eq. (2) leads to a feedback between the flow and the angle ψ that eventually balances and leads to a steady profile in which ψ varies nonmonotonically [Fig. 6(f)].

We compare our results obtained by numerically solving the dynamic equations (2) and (3) to solutions of the steady-state equation for the angular component of the polarization, when assuming rotational invariance and $|\mathbf{p}| = 1$. Under these assumptions, it reads [23]

$$h_{\perp} [4\bar{\eta} + (\nu^2 + 1 - 2\nu \cos 2\psi)] = \sin 2\psi (\nu \cos 2\psi - 1)(\tilde{\zeta} + \nu\bar{\lambda}), \quad (\text{B1})$$

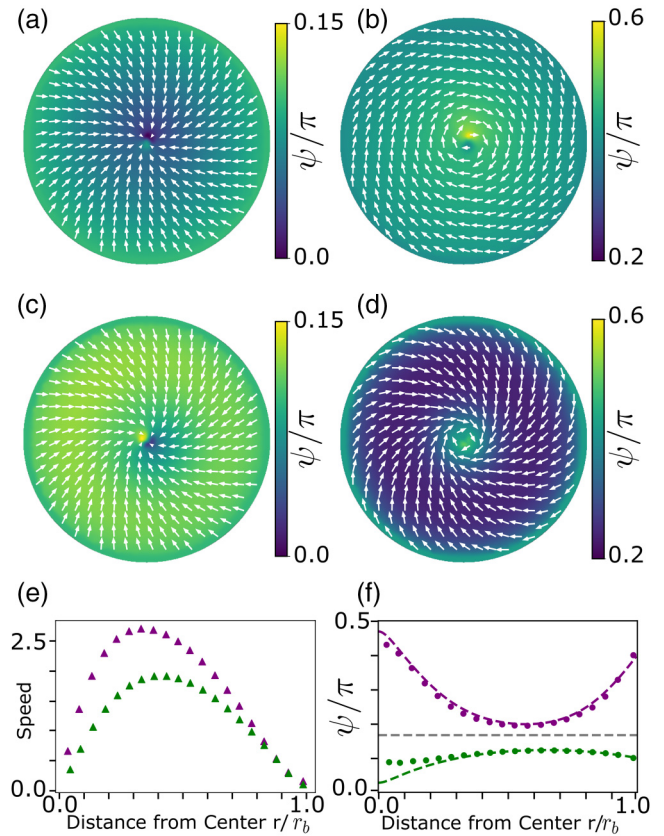


FIG. 6. Steady states for $\bar{\lambda} = 0$, $\Delta\mu = 0.8$. Polarization fields for $\overline{dK} = 1$, $\tilde{\zeta} = 0$, and $\psi_b = \pi/10$ [panel (a)], $\overline{dK} = -1$, $\tilde{\zeta} = 0$, and $\psi_b = 2\pi/5$ [panel (b)], $\overline{dK} = 1$, $\tilde{\zeta} = -1.6$, and $\psi_b = \pi/10$ [panel (c)], and $\overline{dK} = -1$, $\tilde{\zeta} = -1.6$, and $\psi_b = 2\pi/5$ [panel (d)]. White arrows: $\mathbf{p}$; colors: polarization angle ψ/π . (e) Azimuthal average of the angular speed ω for panels (c) (green) and (d) (purple). (f) Azimuthal average of the angle ψ for panels (c) (green) and (d) (purple). Green and purple dashed lines: solutions to Eq. (B1); gray dashed line: Leslie angle ψ_L [Eq. (B3)].

where

$$h_{\perp} = (1 + \overline{dK} \cos^2 \psi) \left[\psi'' + \frac{\psi'}{r} \right] - \frac{\overline{dK}}{2} \sin 2\psi \left[\frac{1}{r^2} + \psi'^2 \right]. \quad (\text{B2})$$

Here, $\psi = \psi(r, t)$ and $\psi' = \partial_r \psi$. In the case $\overline{dK} = 0$ and assuming free anchoring at the boundary, such that ψ_b adapts to the bulk, $h_{\perp} = 0$ and the polarization assumes the Leslie angle ψ_L , with

$$\cos(2\psi_L) = \frac{1}{\nu}, \quad (\text{B3})$$

if $|\nu| \geq 1$. In the case of strong anchoring, that is, when imposing ψ_b and $\partial_r \psi(0, t) = 0$, the numerical solution of Eq. (B1) agrees well with the steady-state solution obtained by solving the full dynamic equations [Fig. 6(f)]. Far from the boundaries, the polarization angle ψ approaches the Leslie angle ψ_L .

APPENDIX C: STOKES FLOW FOR AN ARBITRARY FORCE FIELD

To compute the flow for a given polar texture, we follow a methodology similar to Ref. [39]. At low Reynolds number, the flow field $\mathbf{v}$ satisfies the Stokes equation

$$\eta \Delta \mathbf{v} = \mathbf{f}, \quad (\text{C1})$$

where $\mathbf{f}$ is in our case given by $\nabla \cdot (\sigma^a + \sigma^r)$.

We compute the force density $\mathbf{f}$ numerically on a 256×256 grid. The flow field is then given by the convolution of the two-dimensional Oseen tensor with the force

density

$$v_\alpha(\mathbf{r}) = \iint G_{\alpha\beta}(\mathbf{r} - \mathbf{r}') f_\beta(\mathbf{r}') dA', \quad (\text{C2})$$

where the two-dimensional Oseen tensor is

$$G_{\alpha\beta}(\underline{r}) = \frac{1}{4\pi\eta} \left[[\ln(\mathcal{L}/r) - 1] \delta_{\alpha\beta} + \frac{r_\alpha r_\beta}{r^2} \right]. \quad (\text{C3})$$

Here, $\mathcal{L}$ is a length scale that can be adjusted to give the correct boundary behavior. This is further augmented by a homogeneous solution to the Stokes equation to satisfy the no-slip boundary condition [40].

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