Discretization-Corrected PSE Operators for Particle Methods

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In Lagrangian particle methods for flow simulations, such as smoothed particle hydrodynamics (SPH) or vortex methods, the continuous flow fields are discretized over scattered particle locations. In order to accurately and efficiently compute spatial derivatives of any order, Eldredge et al.[1] proposed a generalization of the original particle strength exchange (PSE) operators[2]. Both the original and the generalized PSE operators involve two numerical approximation errors: the mollification error and the discretization error. In order to minimize the mollification error, the size of the operator support should be as small as possible, whereas a small discretization error is achieved by increasing the number of particles within the operator support. This inherent trade-off hampers the convergence of PSE operators, since the constant discretization error starts to dominate in high-resolution simulations (Fig. 1, right panel).

Discretization correction (DC) is available to restore convergence over the entire range of resolutions. We present a formal framework for DC PSE operators[3], where an additional polynomial correction function ensures that the discretization error vanishes, independently of the kernel support (Fig. 1, left panel). We show that the resulting DC PSE operators are mathematically consistent, even on irregular particle distributions and near boundaries. Moreover, we make explicit the relationships between DC PSE operators, finite difference (FD) stencils, and the operators used in SPH and reproducing kernel particle methods. In particular, we prove that DC PSE operators become equivalent to classical FD stencils in certain limits.

We discuss the numerical properties of DC PSE operators and the role of remeshing in practice by analyzing advection-diffusion problems at various Péclet numbers. We show that the accuracy and efficiency of Lagrangian schemes using DC PSE operators can exceed those of Lagrangian FD schemes since they require less frequent remeshing, hence reducing the remeshing error. In these cases, the computational cost of determining the DC kernel correction functions is amortized by the gain in accuracy.

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[2]Degond & Mas-Gallic, *Math. Comput.*, **53**(188), 485 (1989).

[3]Schrader et al., *J. Comput. Phys.* in press (2010). doi:10.1016/j.jcp.2010.02.004.

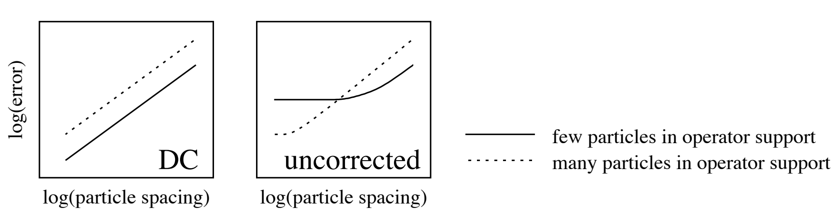

Figure 1: Cartoon convergence plots for PSE operators.